Quantum Circuits II

The Quantum Computing Stack

5	Applications
3	Algorithms
2	Circuits
4	Hardware
1	Mechanics

Now
2^+ Qubits

Last Time
1 Qubit

States and Gates

Covered

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$|-\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

States and Gates

Will Cover

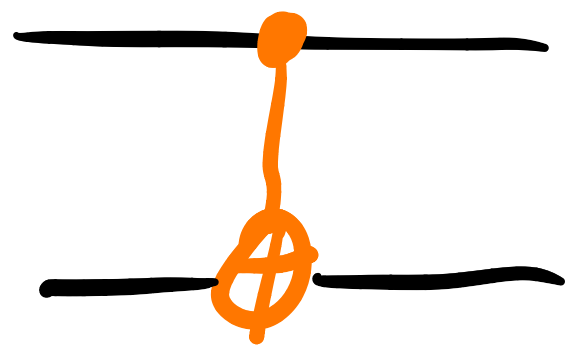
$|01\rangle$

$\vdots$

$|11111111\rangle$

Qubit Binary

CNOT



Entanglement

Today

- Why Unitary?
- Tensor Products
- CNOT Gate

Preskill

Why Unitary?

$H = H^\dagger$

Physics
~~Unsatisfying~~ Answer:

Quantum information transforming by Unitaries is a postulate!

$$i\hbar \frac{\partial \psi}{\partial t} = \hat{H}\psi \Rightarrow U = e^{-i\frac{t}{\hbar}\hat{H}}$$

hermitian

"Hamiltonian"

$$|\psi(t)\rangle = U|\psi(0)\rangle$$

Why Unitary?

Logic Answer:

Quantum gates must be reversible

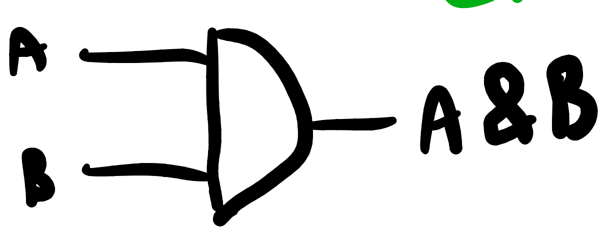
$H = H^\dagger$

$UU^\dagger = I$ energy$\downarrow$

All Unitaries can be inverted.

classical

energy$\uparrow$



A	B	A&B
0	0	0
0	1	0
1	0	0
1	1	1

$n \times n$ mat.

Unitary = Orthonormal Matrix

Rank (n)

$|\text{rows}| = 1 = |\text{columns}|$

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad |-\rangle = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$H|0\rangle = |+\rangle \qquad H|-\rangle = |1\rangle$$

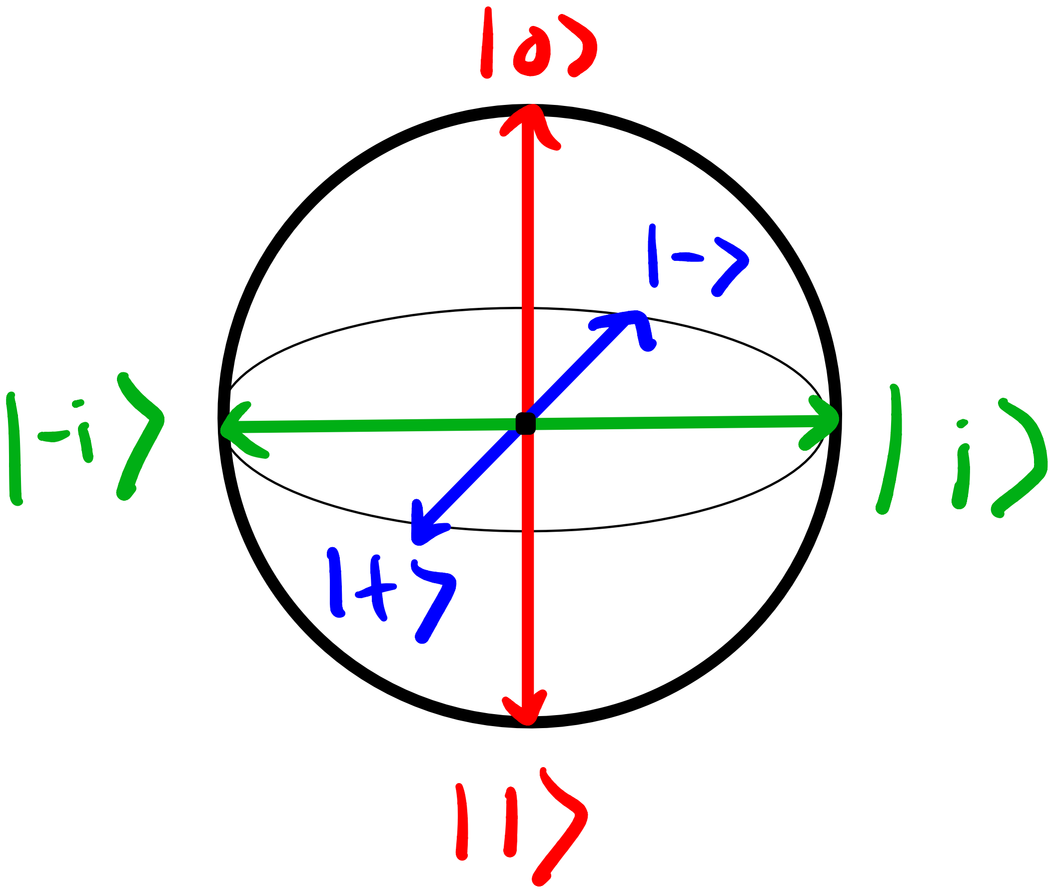
$$H|+\rangle = |0\rangle \qquad H|1\rangle = |-\rangle$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{tikzcd}[column sep=huge, row sep=huge]
|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \arrow[r, leftrightarrow] \arrow[d, leftrightarrow, "H"'] & |-\rangle = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \arrow[d, leftrightarrow, "H"] \\
|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \arrow[r, leftrightarrow] & |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\end{tikzcd}$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Qubit



$$\frac{|0\rangle}{\begin{bmatrix}1\\0\end{bmatrix}} \perp \frac{|1\rangle}{\begin{bmatrix}0\\1\end{bmatrix}}$$

$$\frac{|+\rangle}{\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}} \perp \frac{|-\rangle}{\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}}$$

$$\frac{|i\rangle}{\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}} \perp \frac{|-i\rangle}{\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}}$$

Qubit

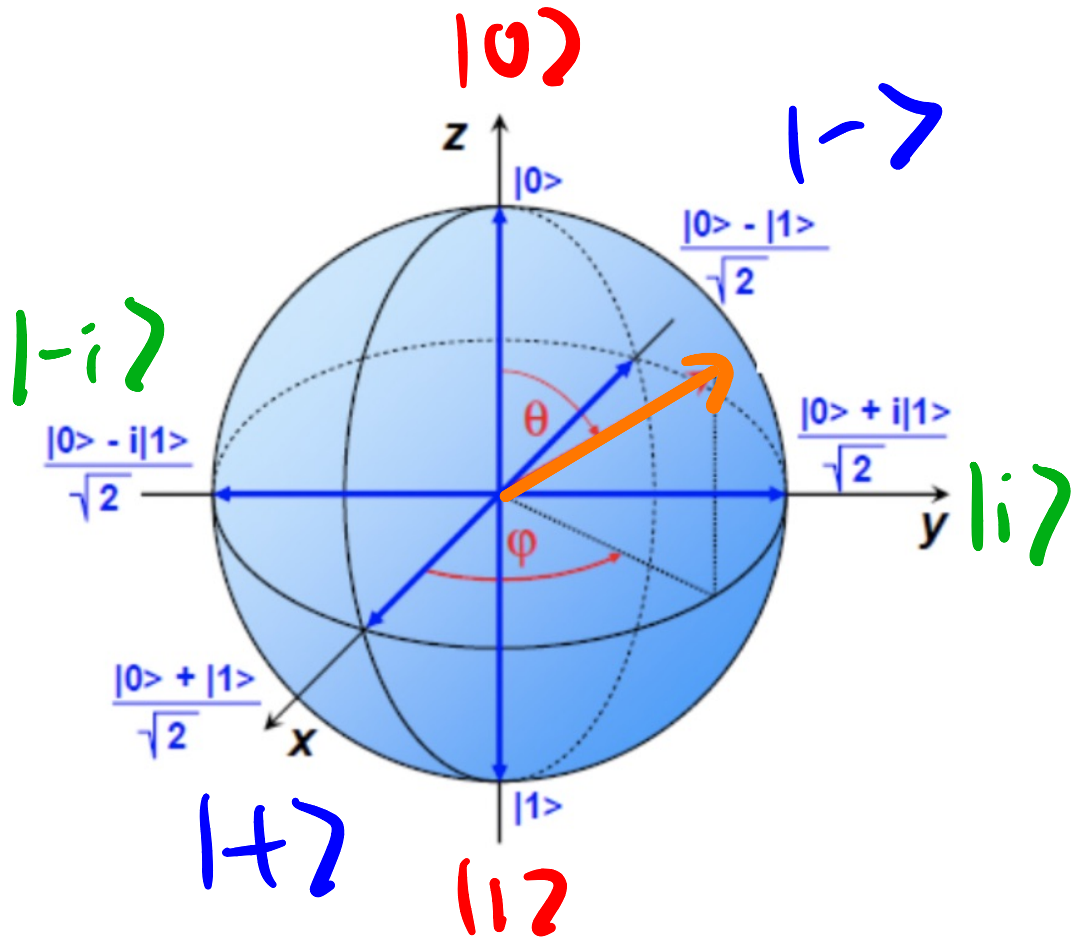
$$|\psi\rangle = \cos(\theta/2)\,|0\rangle + e^{i\phi}\sin(\theta/2)\,|1\rangle = \begin{pmatrix} \cos(\theta/2) \\ e^{i\phi}\sin(\theta/2) \end{pmatrix}$$

Rotation matrices:

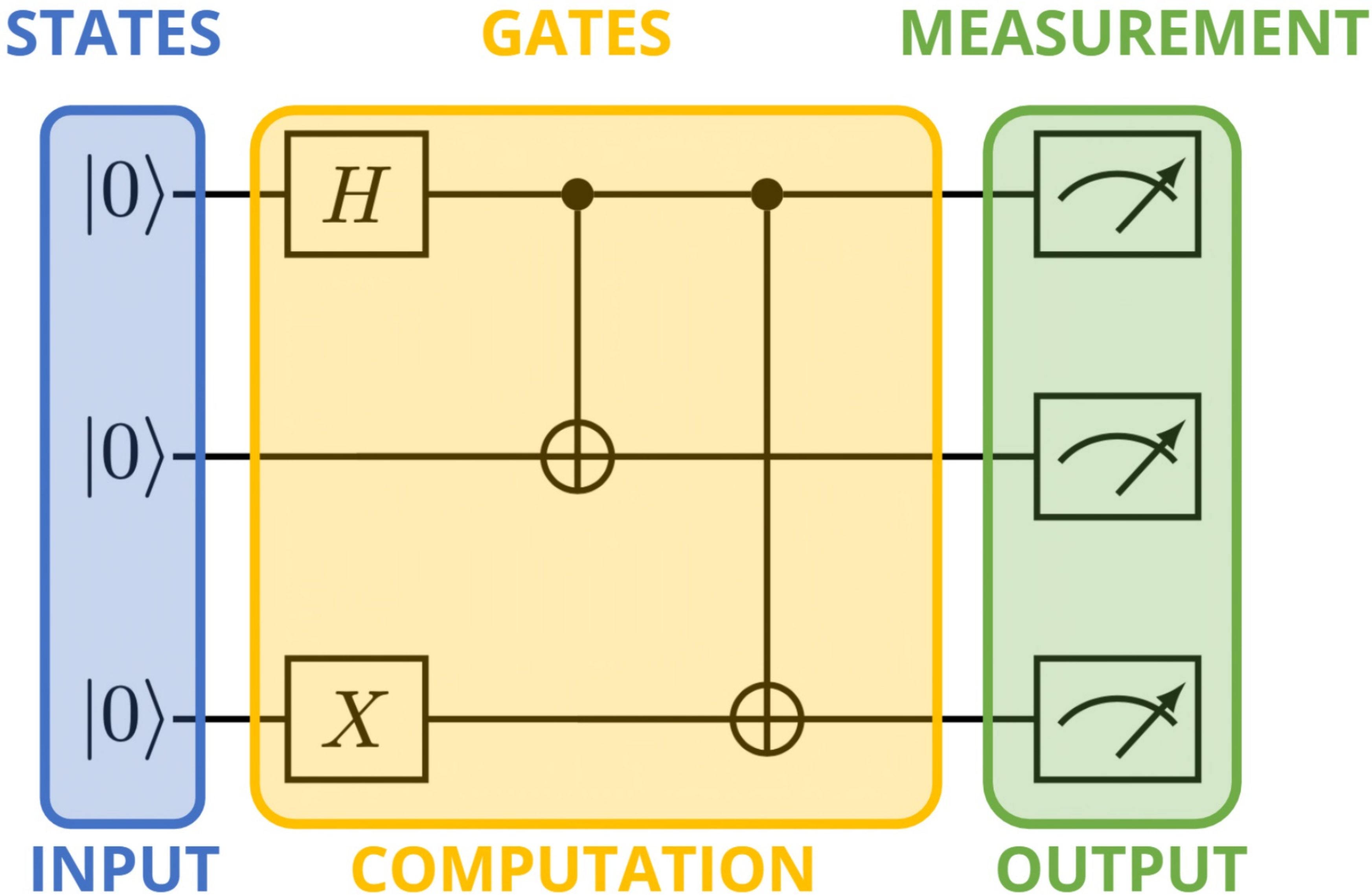
$$\hat{R}_x(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

$$\hat{R}_y(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$$

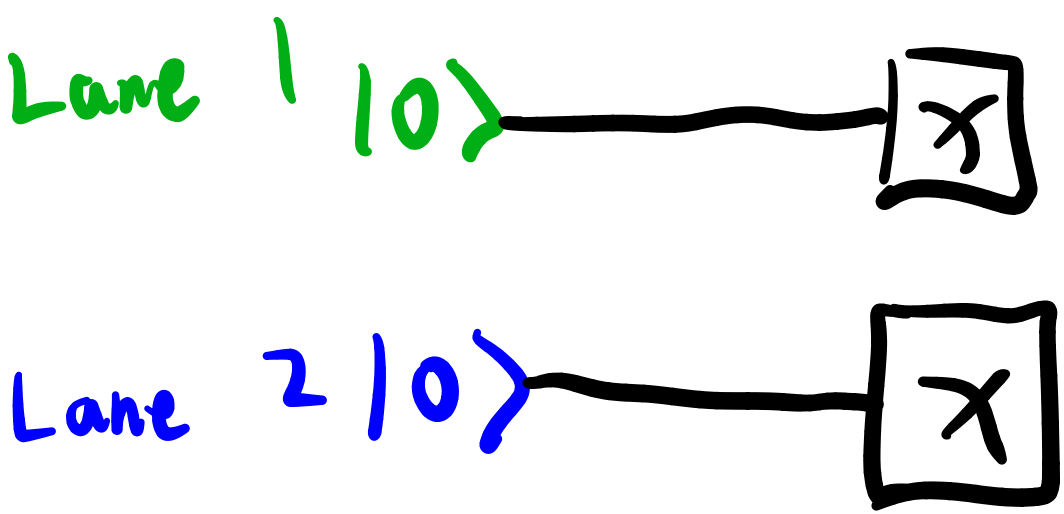
$$\hat{R}_z(\theta) = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$$



Quantum Circuit



Tensor Products of States and Gates represent multiple Qubits



Tensor product

$|0\rangle \otimes |0\rangle$

$\uparrow = \rightarrow$

convention

Tensor Product of States

$$|1\rangle \otimes |0\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ 1 \cdot \begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix}$$

$$|1\rangle = \begin{array}{cc} \begin{bmatrix} 0 \\ 1 \end{bmatrix} & \begin{array}{c} 0 \\ 1 \end{array} \end{array}$$

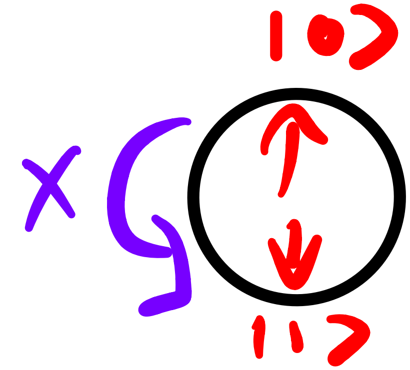
$$|0\rangle = \begin{array}{cc} \begin{bmatrix} 1 \\ 0 \end{bmatrix} & \begin{array}{c} 0 \\ 1 \end{array} \end{array}$$

$$= \begin{array}{cc} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} & \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \end{array}$$

$$= |10\rangle$$

Tensor Product Practice

$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\quad$ $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



$$|0\rangle \otimes |0\rangle = \begin{bmatrix} 1\begin{bmatrix} 1 \\ 0 \end{bmatrix} \\ 0\begin{bmatrix} 1 \\ 0 \end{bmatrix} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad |00\rangle$$

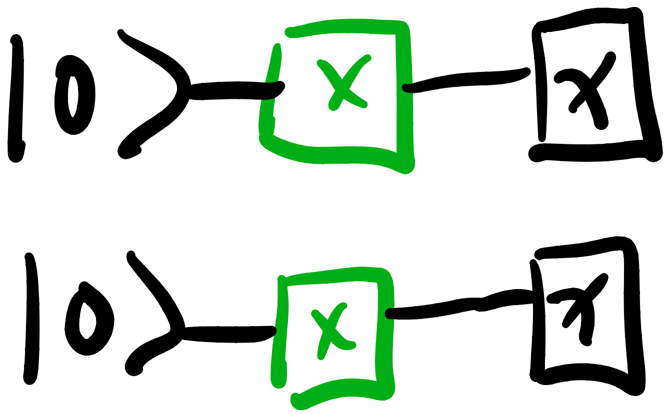
$$|0\rangle \otimes |1\rangle = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad |01\rangle$$

$$|1\rangle \otimes |0\rangle = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad |10\rangle$$

$$|1\rangle \otimes |1\rangle = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad |11\rangle$$

Keyword: Euphoria

Tensor Product of Gates



$$X \otimes X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

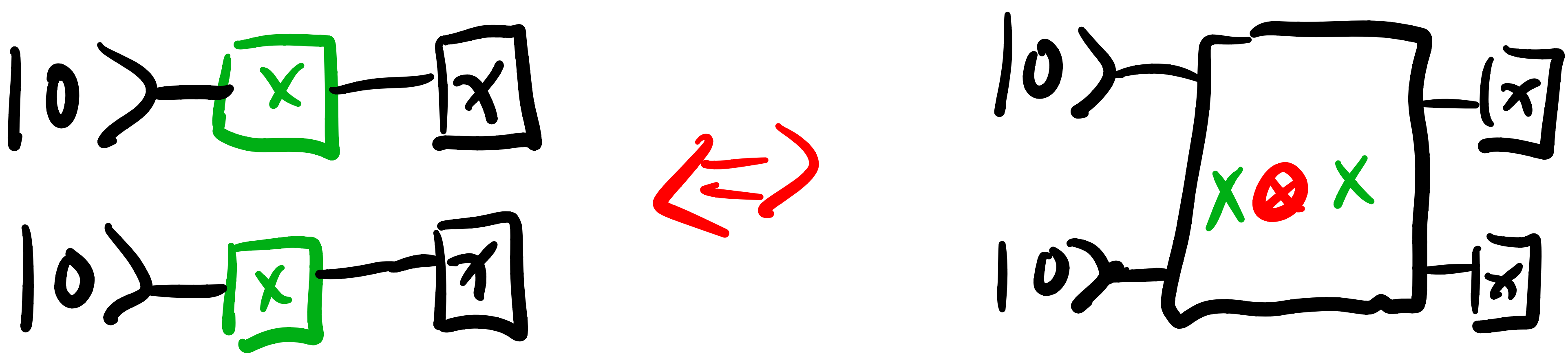
Tensor Product of Gates

$$\begin{array}{cc} \text{in:} & \begin{array}{cc} |0\rangle & |1\rangle \end{array} \\ X = & \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \\ \text{out:} & \begin{array}{cc} |1\rangle & |0\rangle \end{array} \end{array} \qquad \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{array}{cc} \text{in:} & \begin{array}{cccc} |00\rangle & |01\rangle & |10\rangle & |11\rangle \end{array} \\ X \otimes X = & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = X^{\otimes 2} \\ \text{out:} & \begin{array}{cccc} |11\rangle & |10\rangle & |10\rangle & |00\rangle \end{array} \end{array}$$

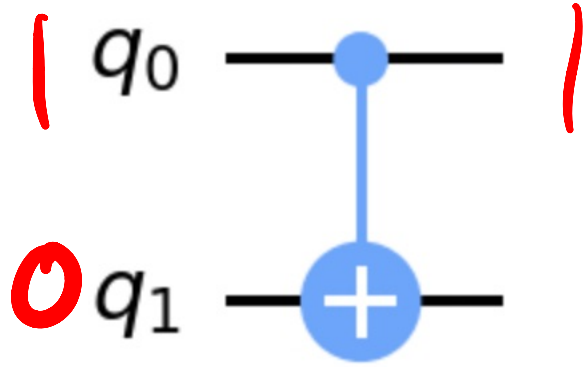
Making Useful

Multi-Qubit Gates



CNOT Gate

$|q_1 q_0\rangle$



- 2 qubit operation
- Matrix Representation*

$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Highly dependant on certain basis

If $q_0 = 0$:

$$q_1 = q_1$$

If $q_0 = 1$:

$$q_1 = (q_0 + q_1) \bmod 2$$

$$CNOT = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$CNOT\,|\overset{q_1}{0}\overset{q_0}{0}\rangle = |\overset{q_1}{0}\overset{q_0}{0}\rangle$$

$$CNOT\,|01\rangle = |11\rangle$$

$$CNOT\,|10\rangle = |10\rangle$$

$$CNOT\,|11\rangle = |01\rangle$$

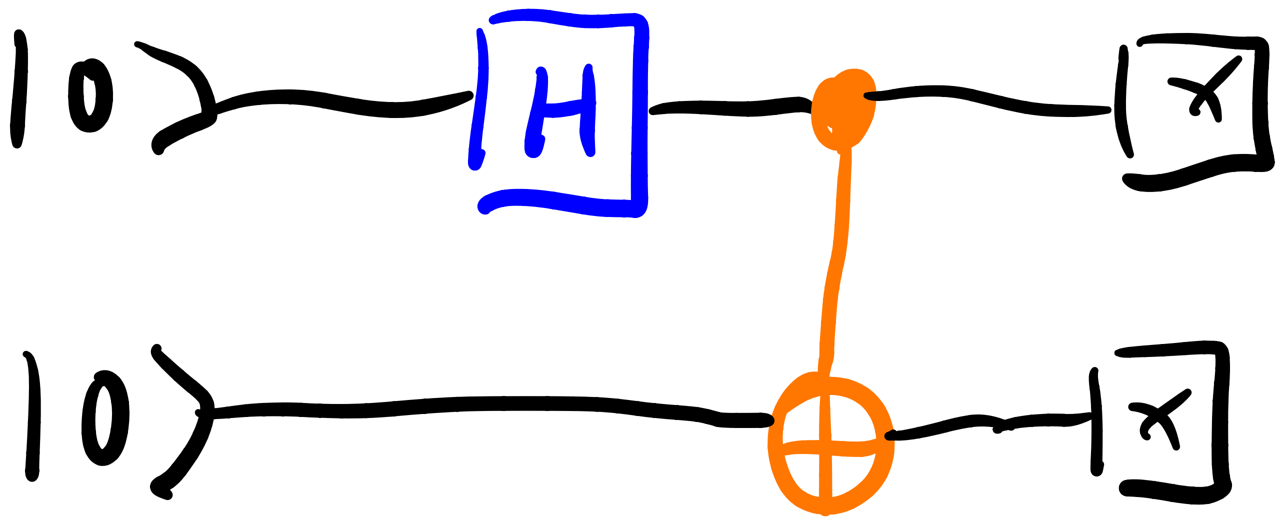
expect $|11\rangle$

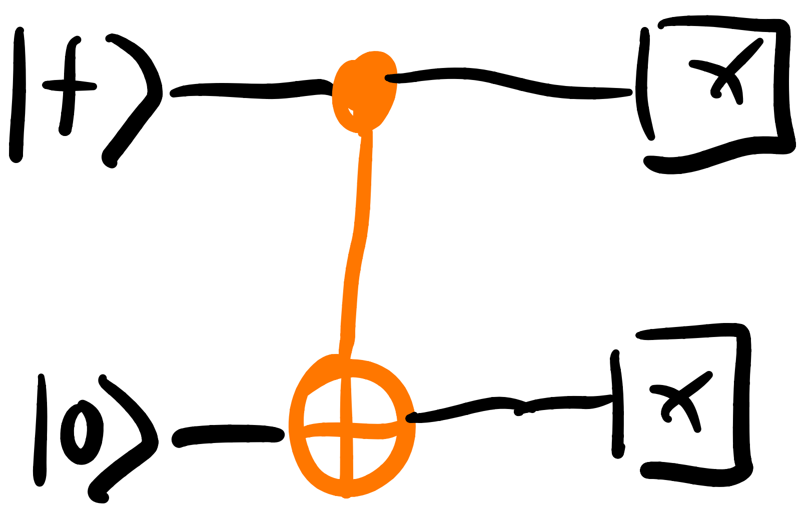
$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} |01\rangle$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

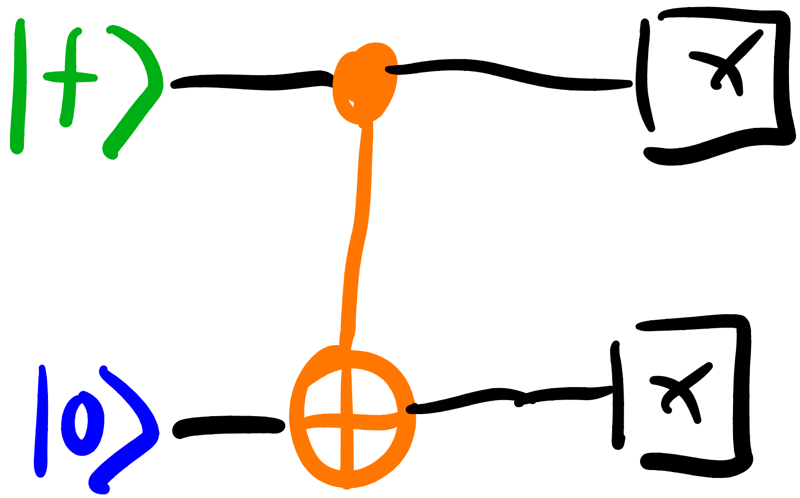
$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = |11\rangle$$

Entanglement





Entanglement



$$\mathrm{CNOT}|0+\rangle$$

$$= \mathrm{CNOT}\frac{(|00\rangle + |01\rangle)}{\sqrt{2}}$$

$$|0\rangle \otimes \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$= \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$

Qiskit Textbook